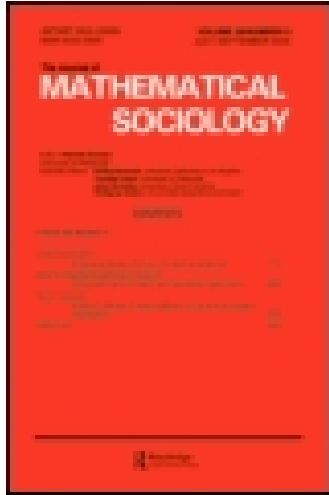


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When Do Matthew Effects Occur?

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When Do Matthew Effects Occur?

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What are the boundary conditions of the Matthew Effect? In other words, under what circumstances do initial status differences result in highly skewed reward distributions over the long run, and when, conversely, is the accumulation of status-based advantages constrained? Using a formal model, we investigate the fates of actors in a contest who start off as status-equivalents, produce at different levels of quality, and thus come to occupy distinct locations in a status ordering. We build from a set of equations in which failing to observe cumulative advantage seems implausible and then demonstrate that, despite initial conditions designed to lead inevitably to status monopolization, circumstances still exist that rein in the Matthew Effect. Our results highlight the importance of a single factor governing whether the Matthew Effect operates freely or is circumscribed. This factor is the degree to which status diffuses through social relations. When actors' status levels are strongly influenced by the status levels of those dispensing recognition to them, then eventually the top-ranked actor is nearly matched in status by the lower-ranked actor she endorses. In contrast, when actors' status levels are unaffected by the status levels of those giving them recognition, the top-ranked actor amasses virtually all status available in the system. Our primary contribution is the intuition that elites may unwittingly and paradoxically destroy their cumulative advantage beneath the weight of their endorsements of others. Consequently, we find that the Matthew Effect is curtailed by a process that,

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| *at least in some social settings, is a property of status itself—its propensity to diffuse through social relations. Implications for future research are discussed.*

Keywords: leadership, social networks, status

1. INTRODUCTION

When Newton and Leibniz independently discovered calculus in the late seventeenth century, Newton's status in the Royal Society enabled him to overshadow Leibniz and receive much of the credit for the breakthrough. Although Leibniz's work was published before Newton's and is the more direct antecedent of modern differential calculus, Newton's prestige in the British scientific community meant that for decades recognition flowed disproportionately in his direction (Ball, 1960, pp. 360–362; Boyer, 1968, pp. 451–452). Nearly two centuries later, when van't Hoff and Le Bel separately discovered tetrahedral carbon, status again affected the allocation of esteem. Le Bel was a marginal player, and his concurrent discovery is likely known only because van't Hoff, a prominent figure in scientific circles, chose to mention it (Ramberg, 2003).

Uniting these examples is a process described by Merton (1968) as the Matthew Effect. When Merton first used the concept in the sociology of science, he drew an important distinction by applying it at two levels, micro and macro. At the micro level, he meant it to describe the process by which high-status scientists get discernibly more credit for comparable intellectual achievements than their less prestigious colleagues. At the macro level, he used it to describe the process, often referred to as cumulative advantage, by which high-status scientists enjoy positive feedback between intangible and tangible resources, and thus eventually gather up a disproportionate share of the rewards.¹ Thus, more abstractly, the concept denotes (i) the principle that, for a given level of quality, higher-status actors enjoy greater payoffs than their lower-ranked counterparts, and (ii) the process by which elite actors amplify their status and collect ever-larger advantages.

Since Merton's influential article, the Matthew Effect has been used as a lens through which to examine a number of topics,

¹For the micro-level version of the Matthew Effect, see Merton (1968, pp. 57–58), and for the macro-level version, see Merton (1968, p. 62). The term was derived from Saint Matthew's Gospel (25:29): "For unto every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken away even that which he hath."

including political legitimacy (Richards, 1968), intraorganizational power (Kanter, 1977), career attainment (Allison and Stewart, 1974; Broughton and Mills, 1980), education (Walberg and Tsai, 1983), healthcare (Link and Milcarek, 1980), litigation (Cooney, 1994), textbook publishing (Levitt and Nass, 1989), Internet publishing (Fleming, Simcoe, and Waguespack, 2008), household income (Dasgupta, 1995), workers' productivity (Weiss, 1984), certification contests (Rao, 1994), organizational growth (Podolny and Phillips, 1996), and economic development (Yang, 1990). It is thus well understood that the Matthew Effect is central among the dynamics that generate social and economic inequality.

Yet it is equally apparent that few competitive systems permit status-based advantages to grow unabated until rewards are nearly monopolized by a single elite. At work instead in most contexts are factors that restrict growth in disparities between elites and their marginal counterparts. As Merton (1988) noted: "Conceived of as a locally ongoing process and not as a single event, the practice of giving unto everyone that hath much while taking from everyone that hath little will lead to the rich getting forever richer while the poor become poorer. Increasingly absolute and not only relative deprivation would be the continuing order of the day. But as we know, things are not as simple as all that" (p. 610).

Using Merton's observations as our point of departure, in this article we extend existing work in sociological theory on status-based attainment by proposing a model of the Matthew Effect that allows us to better understand its boundary conditions. We develop a formal approach that specifies the conditions under which initial status differences result in highly skewed distributions over the long run, and those under which, conversely, the accumulation of status-based advantages is constrained. The question we address is therefore the following: *What reins in the Matthew Effect?* That is, given that the Matthew Effect is almost invariably at work at the micro level, what restricts its realization at the macro level, keeping elites from soaking up almost all available rewards?

Although several scholars, including Merton (1988), have observed factors restraining the operation of the Matthew Effect, much of this work has limited its attention to predetermined, rather than emergent, constraints.² In speculating about what reduces otherwise wide "gaps between the haves and the have-nots in

²An important exception is Podolny's (1993) compelling discussion of emergent status-erosion as the consequence of a high-status organization's choice to expand in market share by encroaching on a lower-status competitor's niche.

science [and] in other domains of social life,” Merton (1988, p. 606) drew attention first to resource-related limitations. In particular, he emphasized restrictions on academic institutions’ capacities to expand and accommodate the stars of young cohorts with new professorships. In addition, he underscored the redistributive tendencies of governments, observing that the value placed on equality within a particular society may cause surplus to be redistributed in a manner that prevents any one laboratory or university from pulling too far ahead of the rest of the field (Merton, 1988, p. 619).

Luhmann (1987) stressed a similar constraint on elites’ opportunities to multiply their resources without bound. According to his historical account, “[p]ride became a sin not only for theologians, and the snob was invented to make sure that in spite of all decline in religion and morality there remained at least one sin that could not be forgiven. At the end of the eighteenth century, it became possible even to boast of low birth to prove one’s capacity to overcome obstacles” (Luhmann, p. 120). Thus, in any given status-contest (at least in many modern societies), a strong appreciation for mobility and turnover necessarily places limits on cumulative advantage.

Collins’s (2000, pp. 29–33) conception of deference as inherently local and situational points to another predetermined limitation on the Matthew Effect. According to his theory, status accrual is particularly likely to occur in networks of individuals centered on a common base of specialized knowledge. Chances for growth in status are thus constrained by others’ inability to properly appreciate a (local) maven’s worth, and so high-status individuals are often confined to a finite base of focused admirers. According to Collins’s view, the pre-existing opportunity structure for status growth is often deeply fragmented and thus dead-ended. Consequently, according to this account, as in Merton’s and Luhmann’s, a predetermined factor counteracts the Matthew Effect.³

³Zuckerman and Kim’s (2003) discussion of low-status consumers’ distaste for high-status producers for fear of the latter’s lack of commitment to the former offers an intriguing additional constraint on the Matthew Effect: “consumers may sometimes regard high-status entrants as *overqualified* for low-status niches—that is, they are rejected because consumers are skeptical that they will be sufficiently committed to serving the niche. Put more prosaically, high-status producers who are perceived as ‘slumming’ or ‘carpet-bagging’ are likely to be rejected” (p. 30). The mechanism to which Zuckerman and Kim call attention is a bridle on cumulative advantage that could be discussed as predetermined or emergent—depending on one’s priors about the capacity of locally evolving dispersions in status to alter preferences for interaction across large divides.

We take a different approach by constructing a formal model of status-based competition that does not impose these constraints *ex ante*. We develop a model in which actors *always* experience the Matthew Effect at the micro level; that is, higher-status actors invariably get greater rewards (and at lower cost) for a given level of quality produced. We then ask: When do these actors *not* collectively produce a Matthew Effect at the macro level? When, in other words, is the status-based monopolization of rewards circumscribed? We therefore build from a set of equations in which failing to observe cumulative advantage seems implausible and then demonstrate that, despite initial conditions designed to lead inevitably to status monopolization, circumstances still exist that rein in the Matthew Effect. We believe that understanding the nature of these circumstances is important both for empirical researchers seeking to construct sampling frames suitable for assessing status effects and for theorists formulating models of competition for intangible rewards.

Our model extends prior work on the Matthew Effect by considering a hypothetical cohort of actors who begin as status equivalents and then progressively occupy distinct positions in a status hierarchy. The set of actors to which we refer would correspond empirically to virtually any group whose members are vying for each other's recognition or respect, for instance, technologists working jointly in an R&D facility or managers debating strategies in a series of meetings. We situate these actors in a dynamic model in which they contribute at different levels of quality, receive varying degrees of recognition from one another as a consequence, and thus eventually occupy distinct roles in a status ordering. We then bring relief to the conditions under which the most able actor enjoys the benefits of the Matthew Effect, assuming virtually all status in the system, and those under which, conversely, the self-reinforcing dynamic of the Matthew Effect is curtailed.

The results of our model highlight the importance of a single factor governing whether the Matthew Effect operates freely or is circumscribed. This factor is *the degree to which status diffuses through social relations*. When actors' status levels are strongly influenced by the status levels of those dispensing recognition to them (i.e., status diffusion occurs), then in due course the top-ranked actor is nearly matched in status by the actor she endorses. By contrast, when actors' status levels are unaffected by the status levels of those recognizing them (i.e., status diffusion fails to occur), the top-ranked actor then collects nearly all status present in the system. Our primary contribution is the intuition that elites may unwittingly and paradoxically destroy their cumulative advantage beneath the

weight of their endorsements of others. Consequently, we find that the Matthew Effect is curtailed by a process that, at least in some social settings, is a property of status itself—its propensity to diffuse through social relations.

We will later portray this constraint on cumulative advantage formally and discuss its implications for empirical research, but we can illustrate our main intuition at this juncture with reference to a well-known contest in the history of American business. Consider, in particular, the power struggle between Steve Jobs and John Sculley at Apple Computer (Carlton, 1997). Jobs, with charisma to spare but searching for a corporate mentor, had successfully persuaded Sculley to leave a promising track at Pepsi and join him at Apple to “change the world.” Jobs’s extraordinary status rapidly magnified Sculley in the Apple fold. Yet, after endorsing Sculley, Jobs eventually found himself overtaken in stature by the former, as Sculley convinced Apple’s board of directors in a moment of crisis to strip Jobs of his power. Thus, Jobs was (at least for several years) surpassed in status by the one he had anointed to assist him. Other corporate leaders are thought to have been more calculating, ousting heirs-apparent before being overtaken by them. Widely discussed examples include James Robinson’s obstruction of Sandy Weill’s route to CEO at American Express; Weill’s subsequent ejection of Jamie Dimon, his longtime protégé and heir apparent at Citigroup (Khurana, 2002, pp. 6–11); and Michael Eisner’s ousting of Jeffrey Katzenberg and Michael Ovitz at Disney. Yet these cases highlight the same theme—that growth in status can be arrested by a feature of status itself—its tendency to diffuse through social relations.

We turn next to the components of our model, which will allow us to examine the boundary conditions of the Matthew Effect. We build our equations from the observations of earlier research on status-based competition (Podolny, 2005). We then discuss our results before concluding with a consideration of how our results might inform empirical research on status in organizational contexts.

2. THE COMPONENTS OF THE MODEL

2.1. Measuring Status

The model consists of n actors in a contest for status. We depict actors’ status levels using Bonacich’s (1987) measure, which has two main advantages. First, it is consistent with a conception of each actor’s status as a function of the recognition received from others in the system—a conception that accords with a definition of

status as a “stock” constructed from corresponding “flows” of respect (Parsons, 1963; Podolny and Phillips, 1996, pp. 454–455; Merton, 1988, p. 620). That is, we define status as an intangible property built from incoming streams of recognition, and we define recognition broadly to include such inputs as respect, esteem, endorsement, commendation, approval, liking, honor, and support.⁴

Second, Bonacich’s (1987) measure permits us to adjust the extent to which actors’ status levels hinge on the status levels of their contacts, that is, to vary the degree of status diffusion in the system. Consider two extremes: At one extreme, a focal actor’s status is just the sum of the acts of recognition shown by her immediate contacts; at the other, the focal actor’s status is shaped not just by the recognition allocated by her contacts but also strongly influenced by her contacts’ own levels of status. A case of such influence, or status diffusion, is in the well-known account of Baron de Rothschild’s endorsement of a friend who requested a loan: “Reputedly, the great man replied, ‘I won’t give you a loan myself; but I will walk arm-in-arm with you across the floor of the Stock Exchange, and you soon shall have willing lenders to spare’” (Caldini, 1989). Thus, through Baron de Rothschild’s strategic recognition of his friend, status spread from the former to the latter. Accordingly, our measurement strategy allows us to exploit variation in the extent to which status diffuses, or is instead contained, when status-conferring gestures are made.

We construct for each time period an asymmetric matrix \mathbf{R}_t whose entries record the flow of recognition each actor receives from all other $n - 1$ actors in the system. Cell R_{ijt} records the level of recognition (respect or esteem) actor i obtains from actor j in period t . Using Bonacich’s measure, the status of i at t may be written as:

$$\mathbf{S}_t(\alpha, \beta) = \alpha \sum_{k=0}^{\infty} \beta^k \mathbf{R}_t^{k+1} \mathbf{1} \quad (1)$$

where S_{it} is an element of \mathbf{S}_t denoting the status of actor i at t .⁵ We selected the scaling parameter α so that in each period, regardless of the size of the network, the actor whose entry equals 1 in \mathbf{S}_t does

⁴Very briefly, two additional points merit attention. First, which of these inputs best corresponds to the status-conferring flow we have termed recognition will of course vary by empirical setting. Second, we do not assume that these acts of recognition are necessarily “genuine.” We are agnostic with respect to motive. That is, recognition can be strategic or feigned rather than earnest, and yet prove consequential in W.I. Thomas’s sense.

⁵The solution to Eq. (1) in matrix form is $\mathbf{S}_t(\alpha, \beta) = \alpha(\mathbf{I} - \beta\mathbf{R}_t)^{-1}\mathbf{R}_t\mathbf{1}$, where \mathbf{I} is an identity matrix.

not possess a disproportionately high or low level of status (Bonacich, 1987, p. 1173). We thus selected α so that the squared length of $\mathbf{S}_t(\alpha, \beta)$ equals the number of actors in \mathbf{R}_t permitting meaningful comparisons across systems of different sizes. The diagonal elements of \mathbf{R}_t equal zero, and $\mathbf{1}$ is a column vector of ones.

Of particular importance is our method for determining the parameter β . We define β as follows:

$$\beta = \rho\lambda \quad (2)$$

where λ equals the reciprocal of the largest normed eigenvalue of \mathbf{R}_t and $0 \leq \rho < 1$. Thus, when $\rho = 0$, actors' status levels are determined only by the recognition flowing from their adjacent contacts, and as $\rho \rightarrow 1$ actors' status levels are increasingly determined as well by the status levels of these contacts. The parameter β can be thought of as a diffusion parameter, determining the extent to which status circulates through ties in a network.

This interpretation of β is perhaps easiest to appreciate if Eq. (1) is defined recursively in nonmatrix form as: $S_{it}(\alpha, \beta) = \sum_j (\alpha + \beta S_{jt}) R_{ijt}$ (Bonacich, 1987, p. 1173, Eq. 3). If ρ (and thus β) equal zero, then it is clear that the status of actor i simply reduces to the sum of the levels of recognition going to i from actors j in the columns of \mathbf{R}_t . In this scenario, the distinct levels of status S_{jt} of these other actors have no bearing—status diffusion does not occur, and therefore i 's status equates to i 's row sum (adjusted by the scaling constant α) in \mathbf{R}_t . In contrast, as ρ rises, the status levels of actors j diffuse through their flows of recognition directed at i and thus affect i 's stock of status.

We now turn to the specific steps and corresponding equations by which \mathbf{R}_t (and thus \mathbf{S}_t) from Eq. (1) are updated as $t = 2, 3, \dots, T$, where T is the final period in the contest. In what follows, we describe the equations by which actors produce at optimal levels of quality—specifically, levels that maximize the gap between forecasted future status and anticipated costs—and in return receive recognition that serves as the basis of their future status. Appendix A presents a glossary of symbols used in this and subsequent sections.

2.2. Forecasted Status

We portray actors pursuing status as a reward, following much prior research (Weber, 1946; Blau, 1955; Goode, 1978). In a classic study, Barnard (1938) asserted that nonpecuniary payoffs, such as distinction or status, were frequently more effectual than monetary rewards for the task of growing large organizations. The attractions

of status are also apparent in Merton's (1968) sociology of science, in which he pictured individuals laboring primarily for the respect of their peers. More generally, Park and Burgess (1921) argued that although "men work for wages... they will die to preserve their status" (p. 30). In addition, individuals value status because it plays a part in generating other, more tangible forms of advantage, including information (Hagstrom, 1965; Merton, 1968) and influence (Goode, 1978; Taylor, 1987). Consequently, on account of a general desire for status as an end and as a means, the actors in our model pursue status as their reward.

Our forecasted status function in Eq. (3) has two important features: the anticipation of a micro-level Matthew Effect and an upper bound on the level of future status attainable.

$$S_{F,i,t+1}(Q_{it}) = \left[1 - \frac{1}{S_{it}Q_{it} + 1} \right] S_{\max} \quad (3)$$

Starting with the left-hand side of Eq. (3), $S_{F,i,t+1}(Q_{it})$ denotes the status a given actor anticipates possessing in the future as a function of Q_{it} , the quality of her contribution at time t . We use the subscript F to reflect the fact that status is forecasted, and $t + 1$ because of the gestation period necessary for quality to affect status. Turning to the denominator, we also see status and quality as distinct entities. While we view quality as an outcome that might arise from a "blind taste test" of one's work (Benjamin and Podolny, 1999), we picture status as a relational property affecting how quality is expected to translate into future rewards.

More precisely, in keeping with the denominator in Eq. (3) in which Q_{it} is multiplied by S_{it} , for a chosen level of quality, high-status actors expect greater recognition than their marginal counterparts. Thus, they anticipate benefiting from—or suffering from, if their status is low—a micro-level Matthew Effect, where a high-status actor enjoys a larger payoff for the same contribution than a low-status peer. We thus build on earlier work that, in addition to Merton (1968), has asserted that higher status amplifies rewards for a chosen demonstration of quality. For instance, in his theory of distributive justice, Homans (1961, pp. 234–237) argued that the rewards an individual receives for performing a particular role (e.g., the intangibles that might accrue to a teenager for bowling well with his peers) do not arise solely from the quality of that role performance; the rewards are instead conditional on that individual's larger social standing (Alexander, 1987, pp. 188–189).

Finally, the functional form used in Eq. (3) implies an upper bound on the amount of status a focal actor can expect to realize. Our

motivation for including this term, and thus imposing an asymptote, resides in the fact that status cannot (at least in a system of fixed size) increase without bound. On the contrary, for a system of a given size and set of social rules, there exists some “ideal” state of the world for the top-ranked actor whose objective is to maximize her status. As we discuss further in Appendix B, this state is one where the top-ranked actor exclusively receives recognition from all others in the system (i.e., alters show respect only to the top-ranked actor and to no one else) and the actor on top in turn metes out recognition equally to the others.

We denote the status attainable under this extreme condition by the multiplier S_{\max} . The range on forecasted status, $S_{F,i,t+1}$, is bound by zero—when either S_{it} or Q_{it} equals zero—and S_{\max} —as the denominator in the subtracted term grows large. The maximum level of status may be depicted (as shown in detail in Appendix B) by the following equation, which confirms that S_{\max} is rising in n , the number of actors in the system. Thus, as the market of prospective admirers increases, the size of the status pie correspondingly expands.

$$S_{\max} = [(n-1)/(n^2 + (n-1)(\beta^2 + 2\beta - 3))]^{1/2}(n-1+\beta) \quad (4)$$

Using Eqs. (3) and (4), we display hypothetical forecasted status schedules for two actors in Figure 1, where forecasted status resides on the ordinate and quality is on the horizontal axis. Figure 1 shows greater expected rewards for a given level of quality at higher levels of status. The height of the asymptote, to which the high-status actor’s reward schedule most closely approximates, equals S_{\max} . In our model, actors deploy functions of this type to guide choices about quality produced.

2.3. Forecasted Cost

We assume further that each actor approximates her cost schedule and selects a profit-maximizing level of quality Q_{it}^* . The cost schedule in our model relates expected cost to quality chosen and incorporates multipliers for shared constraints, ability, and status:

$$C_{F,it}(Q_{it}) = kQ_{it}^a A_i^{-b} S_{it}^{-c} \quad (5)$$

$C_{F,it}(Q_{it})$ is the cost actor i expects to incur at time t in the course of producing at quality level Q_{it} . Quality of contribution is therefore the central choice variable. Just as scientists can (within the bounds of their skills, training, and institutional context) determine the level of quality they wish to contribute to a laboratory or seminar series

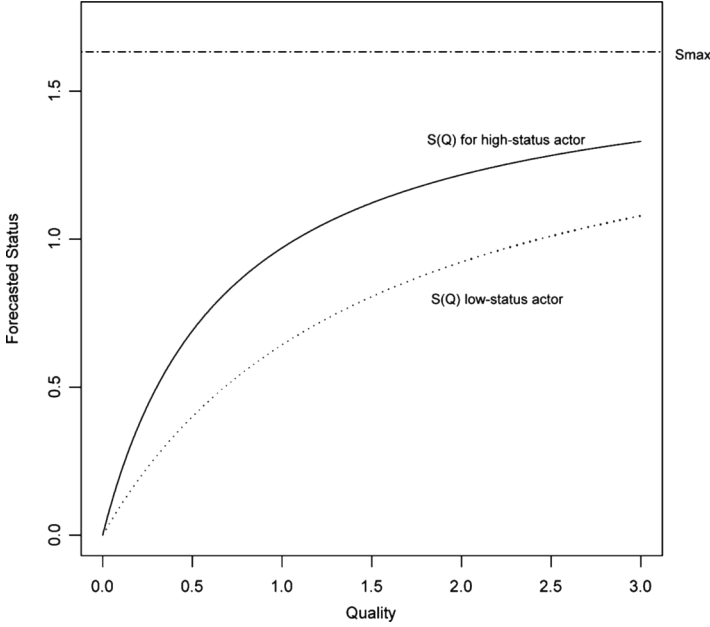


FIGURE 1 The relationship between quality Q_{it} and forecasted status $S_{F,i,t+1}$ for a high- and low-status actor from Eq. (3). A solid line is displayed for the high-status actor’s forecasted status function and a dotted line is used for that of the low-status actor. The status score S_{it} of the high-status actor was set to 1.46, and to .65 for the low-status actor. S_{max} from Eq. (4) was set to 1.63.

in a given timeframe, the actors in our model are selecting a level of quality Q_{it} .

Moving to the right-hand side of Eq. (5), the first term k is a shifter reflecting anticipated generic costs. Among scientists, for instance, k would rise with anticipated increases in the costs of securing research funding. Contouring marginal costs is the exponent a . We begin by setting $a = 2$, so that marginal costs increase with quality, and then in the formal analyses presented in Appendix D we set a equal to unity, which allows for particularly transparent robustness checks of our main results. A_i is the time-constant ability level (which might result from education or prior experience) of actor i . We follow Spence (1974) in attaching shallower cost schedules ($b > 0$) to more able actors (cf. White, 2002). Just as Spence’s more able job candidates complete years of education at lower cost, our most able actor (other factors constant) confronts the gentlest forecasted cost curve.

Status, like ability, also reduces forecasted costs in our model. Here, we follow earlier investigations describing mechanisms by which high-status actors more easily realize their objectives (Merton, 1968, pp. 61–62). Some cost-related advantages thought to accrue to occupants of higher-status positions include easier access to coveted financial and human resources (such as grants and the most talented graduate students in the sciences), as well as greater confidence and more favorable (rather than constraining) expectations from peers (Podolny, 2005; Whyte, 1993, pp. 14–25). Correspondingly, we assume that the expected cost of producing at a given level of quality is

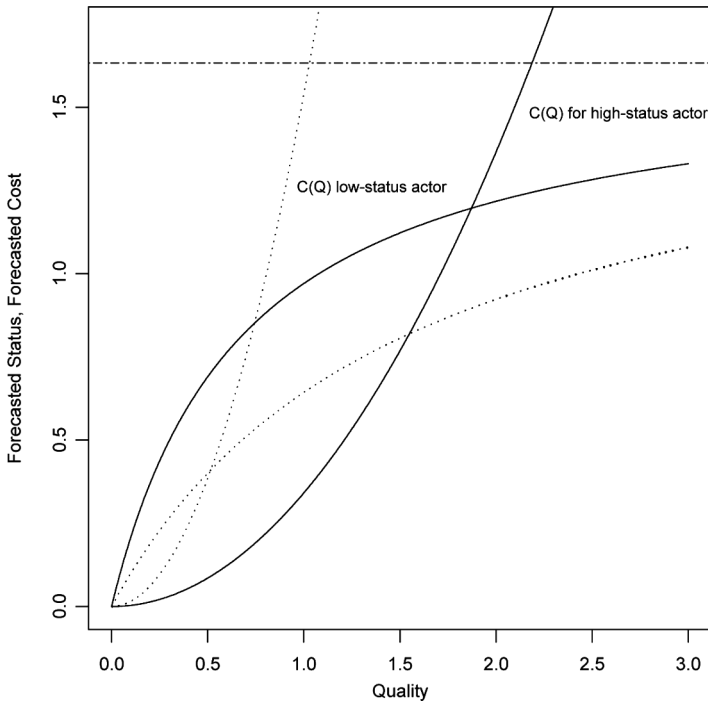


FIGURE 2 The addition of cost curves, $C_{F,it}(Q_{it})$, for the high- and low status actors from Eq. (5). Solid lines are used for the expected status and expected cost functions of the high-status actor and dotted lines are used for those of the low-status actor. With the exception of setting $a = 2$, all parameters were set to unity, so that the high-status actor confronts a gentler cost schedule. In addition, for this example, $A = 1$ for the low-status actor and $A = 2$ for the high-status actor.

lower for higher-status actors, so that the exponent operating on S_{it} is always negative. Thus, in Figure 2 we add two hypothetical cost schedules to the pair of forecasted status functions shown in Figure 1, depicting the negative association between status and cost.

2.4. Deriving Optimal Quality Q_{it}^*

Using Eqs. (3) and (5), we can now derive Q_{it}^* . We start from our earlier claim that each actor opts for a quality level that maximizes the gap between anticipated future status $S_{F,i,t+1}$ and anticipated cost $C_{F,it}$. Consequently (as shown in detail in Appendix C), after setting $\alpha = 2$ from Eq. (5) for simplicity, and setting $\partial S_{F,i,t+1}(Q_{it})/\partial Q_{it}$ from Eq. (3) equal to $\partial C_{F,it}(Q_{it})/\partial Q_{it}$ from Eq. (5), we arrive at the following equation for Q_{it}^* :

$$Q_{it}^* = \left[-\frac{q}{2} + \left[\frac{q^2}{4} + \frac{p^3}{27} \right]^{1/2} \right]^{1/3} + \left[-\frac{q}{2} - \left[\frac{q^2}{4} + \frac{p^3}{27} \right]^{1/2} \right]^{1/3} - \frac{2}{3S_{it}} \quad (6)$$

where $p = -\frac{1}{3S_{it}^2}$, $q = -\left[\frac{S_{\max}}{2\Theta S_{it}} + \frac{2}{27S_{it}^3} \right]$, and $\Theta = kA_i^{-b}S_{it}^{-c}$.⁶

2.5. Computing Future Status

Using Eq. (6) for Q_{it}^* , we can now estimate the recognition matrix for the next time period, \mathbf{R}_{t+1} , from which we compute new status scores \mathbf{S}_{t+1} and the extent to which the Matthew Effect (at the macro level) has been realized. We proceed in two steps. First, we use the following multiplicative equation to estimate \mathbf{R}_{t+1} :

$$r_{ij,t+1} = R_{ijt} \cdot S_{it} \cdot Q_{it}^* \quad (7)$$

where $r_{ij,t+1}$ is the nonnormalized flow of recognition from j to i at $t + 1$. To build in state dependence, R_{ijt} denotes j 's recognition to i in the current time period. S_{it} is the status of i at t , calculated from Eq. (1). Together with dyadic recognition R_{ijt} , S_{it} therefore shapes the effect of quality Q_{it} . Consequently, although the recognition accruing to i from j is affected by i 's level of quality, status acts as a prism

⁶ Q_{it}^* was set to zero in our simulations when any of these conditions was met: $S_{it} = 0$; $Q_{it}^* < 0$; or $q^2/4 + p^3/27 < 0$. The latter, which requires the square root of a negative number, can occur for extremely small values of status (e.g., two actors for whom $S_{it} < 10^{-12}$ met this condition in the simulations used to generate Fig. 3).

(Podolny, 2001) through which quality gets viewed by peers in the process of conferring rewards.⁷

Second, we normalize recognition scores for each column actor j , in keeping with the premise that each column actor j has a budget of 1.00 units of recognition to allocate across the other $n - 1$ actors in the system. We constrain the sum of each column actor's dispensed recognition to unity, thus making each recognition matrix column-stochastic, to reflect an adjustment process that regularly occurs in social life: Individuals who gush over others—those who mete out excessive respect and esteem to their counterparts—generally find that audiences discount their gestures.⁸ Conversely, for those who rarely dispense praise, any act of endorsement on their part, however qualified, gets coded as significant. We build this adjustment process into our model by normalizing each entry by the sum of its column:

$$R_{ij,t+1} = r_{ij,t+1} / \sum_{i=1}^{n-1} r_{ij,t+1} \quad (8)$$

$$\mathbf{R}_{t+1} = [R_{ij,t+1}] \quad (9)$$

2.6. Measuring the Matthew Effect

We then compute both an updated vector of status scores \mathbf{S}_{t+1} by applying Eq. (1) to \mathbf{R}_{t+1} as well as a measure of the extent to which the Matthew Effect has been realized among the n actors contending for status. We use the ratio of the top-ranked actor's status score to the sum of all the status scores in the system to measure the Matthew Effect at the macro level:

$$ME_{t+1} = \max_i (S_{i,t+1}) / \sum_{i=1}^n S_{i,t+1} \quad (10)$$

To the degree that ME_{t+1} approaches unity, the top-ranked actor has been able to collect all available status. Our characterization of the

⁷Although it would be straightforward to add an error term to the recognition function, we maintain the existing formulation for simplicity. In addition, although this decision eliminates a source of the "looseness" of the linkage between quality and status (Podolny, 1993), it has the advantage of making it *harder* to rein in the Matthew Effect. Of course, the Matthew Effect will always fail to unfold in communication systems fraught with extreme noise (cf. Merton, 1968), where current quality—typically made easier to achieve by prior status—will necessarily never strongly affect future status.

⁸To use an example from the academic sphere, this is particularly true for colleagues who have the unfortunate habit of describing each newly-minting doctoral candidate for whom they write as among the top three students they have ever supervised. A fixed effect for such letter writers is implicitly estimated.

Matthew Effect in Eq. (10) as a transparent share-based measure is consistent both with the classic text after which the concept was named—that to “every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken away even that which he hath”—and with Merton’s (1968) discussion of the accumulation of rewards by elites. We turn now to results identifying when the Matthew Effect continues unabated, and when, conversely, social-structural circumstances keep it in check.

3. RESULTS

We begin by portraying the time-varying status scores of a set of actors in Figure 3. To generate these trajectories, we assigned values to our parameters as simply as possible. Starting with β in Eq. (1), which we expressed as the product of ρ and λ in Eq. (2)—where $0 \leq \rho < 1$ and λ denotes the reciprocal of the largest normed eigenvalue of \mathbf{R}_t —we set $\rho = 0$. For Eq. (3), where S_{\max} , the maximum amount of status

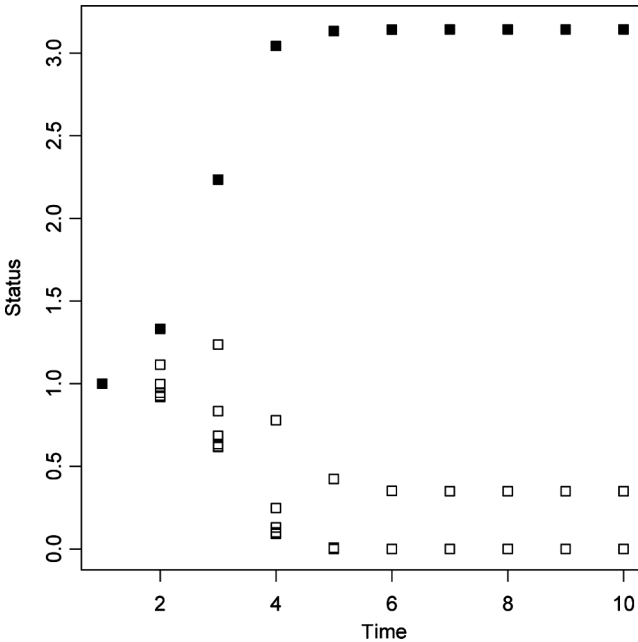


FIGURE 3 The status scores of $n = 10$ actors over ten time periods. These series were generated using the following parameter values: $k = 1$, $a = 2$, $b = 1$, $c = 1$, $v = 2$, and $\rho = 0$. The series for the actor having the most status by the final round is marked accordingly.

attainable, hinges on the number of actors in the system as shown in Eq. (4), we selected $n = 10$.

Turning to Eq. (5), which depicts forecasted cost as a function of quality, we set k , our shifter for generic costs, equal to unity. We fixed $a = 2$ to reflect a perceived rise in marginal cost with quality. We set $b > 0$ and $c > 0$, assigning unity to both exponents, so that cost schedules grow gentler with ability and with status. To assign entries A_i in our vector of ability scores, we proceeded in line with our aim to work from initial conditions that are conducive to large disparities in status and then to identify when the Matthew Effect nonetheless goes unrealized. More specifically, we began with $\mu_i \in [0, 1]$ distributed uniformly, allowed an exponent v to act on μ_i and then added a constant, so that $A_i = \mu_i^{\exp(v)} + 1$ and $A_i \in [1, 2]$. Using this specification permitted us to vary the degree to which the ability distribution is left- or right-skewed. We chose $v = 2$, which makes the distribution thick on the left and thus favors cumulative advantage.

Using these initial parameter values, in Figure 3 we observe the “fanning out” process (Elder, 1969; Dannefer, 1987) characteristic of the Matthew Effect. While at first actors lack a tangible basis for distinguishing among each other, gradations in status materialize as soon as they produce at distinct levels of quality induced by their varied endowments of ability. Viewing these quality-related disparities, each actor updates her appraisal of others, a process that progressively dilates the status hierarchy until the system reaches steady-state (cf. Bothner, Stuart, and White, 2004). Consequently, a feedback loop operates as follows: The ability of each actor determines the initial level of quality she chooses, which then affects the recognition she receives and thus her status; this in turn affects future productivity (and concomitant intangible rewards), further updating others’ perceptions—and so the cycle persists until the status ordering stabilizes. When this steady-state occurs, variations in status far exceed differences in ability, whose levels stay fixed on the $[1, 2]$ interval.⁹

⁹So that actors start the contest as status-equivalents—with status S_{it} equal to unity for all i in the first round—we assigned $1/(n - 1)$ to the off-diagonal entries in the recognition matrix \mathbf{R}_t at $t = 1$. Consequently, the only differentiator at the onset is ability A_i . Variations in ability give rise to distinct levels of quality Q_{it} at $t = 1$ from Eq. (6). After differences in Q_{it} arise, these then bring about differences in flows of recognition in the matrix \mathbf{R}_t at $t = 2$, according to Eq. (7). Therefore, by the second round, actors differ in status. When $t = 1$, the two multipliers preceding Q_{it} in Eq. (7)— R_{ijt} and S_{it} —both necessarily equal unity, exerting no effect until actual acts of dispensing recognition have occurred and differences in status have therefore surfaced by $t = 2$.

Turning to Figure 4, we portray the same process from another angle, now depicting our measure of the Matthew Effect, rather than individuals' status levels, across time. We use the ratio of the top-ranked actor's status score to the sum of all status scores to capture the extent to which the Matthew Effect has been realized, keeping with Eq. (10). The plot in Figure 4 reveals insignificant change in the relative status of the top-ranked actor beyond the sixth round, after which she maintains more than 89% of the status available in the system. We thus observe a trajectory in which the top-ranked actor amasses intangible rewards at a level far greater than her lead over others in ability. While her share of the market for intangibles exceeds 89%, her ability relative to the sum of all endowments of ability, $\max_i(A_i) / \sum_{i=1}^n A_i$, only equals .17.

To assess the validity of our model, we also explore the effects of shifting several of our model's parameters before moving to our primary finding. In particular, we illustrate in Figure 5 the consequences of increasing k , b , c , and n , thus adding four functions

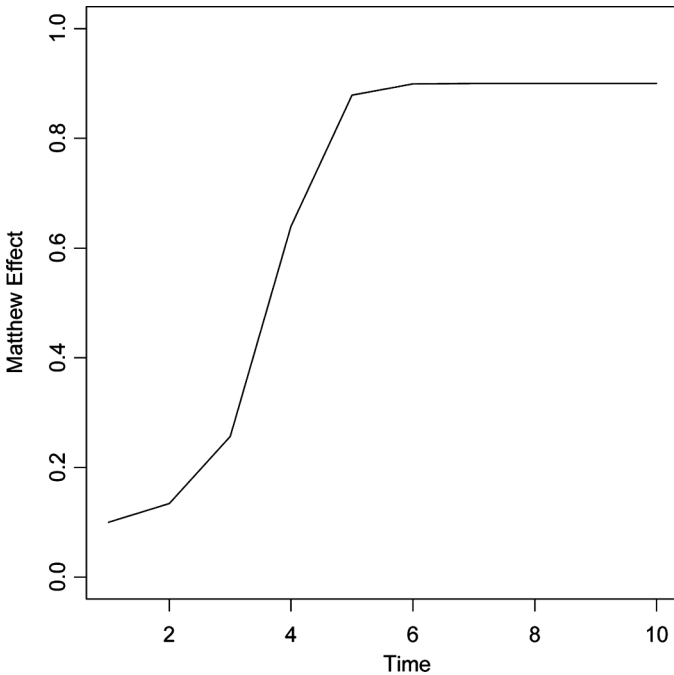


FIGURE 4 The Matthew Effect, as described in Eq. (10), against time, using the same parameter values selected for Figure 3.

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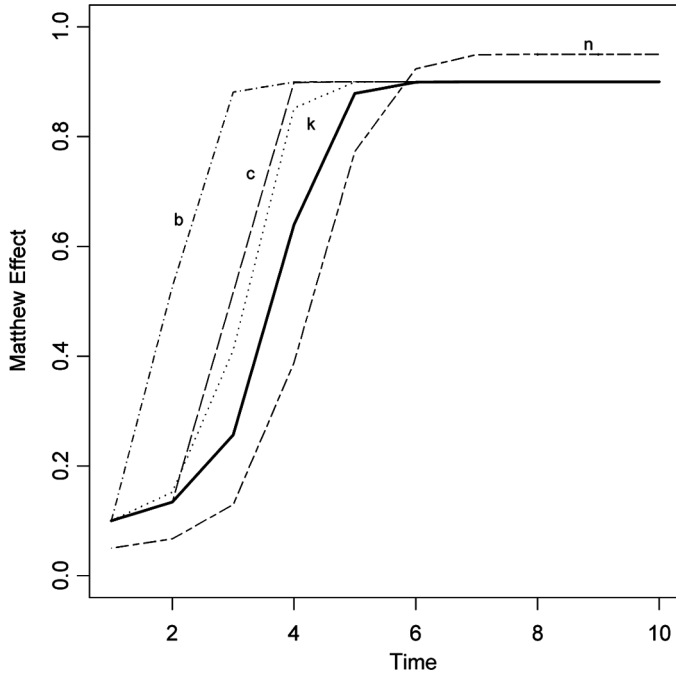


FIGURE 5 Four new pathways of the Matthew Effect, for different parameter values, added around the pathway (in thick solid-line) from Figure 4. The objective of Figure 5 is to illustrate how our measure of the Matthew Effect responds to shifts in the parameters. With the exception of n , which we raise from 10 to 20, all other parameters were raised from 1 to 10. When increasing a particular parameter, we kept all other parameter values at the levels used in Figure 4. We have labeled each new function by the parameter changed.

to the original solid-line function depicted in Figure 4. With the exception of n , which we raise from 10 to 20, we increase parameters from 1 to 10. To ease interpretation of Figure 5, we labeled each function with the new value assigned to the parameter whose value we altered. When shifting upward the value of any particular parameter, the others retain the values we gave them in the course of generating Figures 3 and 4.

Starting with k , which we adjust upward by an order of magnitude, we find that this increase in generic costs favors the top-ranked actor, enabling her to win the contest for status slightly earlier. The dotted-line function corresponding to an increase in k is thus to the left of the solid-line pathway that was first shown in Figure 4. As we raise the

number of actors from 10 to 20, more time is required for the system to reach steady-state. With more competitors in the tournament, time-to-equilibrium expands. Conversely, raising b , the exponent on ability A_i , ushers the top-ranked actor to near status monopolization more quickly. Since ability affects quality Q_{it} immediately, in the first round, raising b has a pronounced effect on the rate at which steady-state is reached. Similarly, as c increases, giving higher-status actors a greater cost advantage, the top-ranked actor arrives at a pinnacle at a discernibly faster rate than in Figures 3 and 4, where c was set only to 1. We thus find a pattern consistent with prior work on the efficiency-related benefits of occupying an ascendant status position. In Appendix D, we present a formal result in keeping with the favorable effect of raising c on the rate of the top-ranked actor's ascent observed in Figure 5.

Whereas the findings in Figure 5 follow predictably from our model's premises and thus validate its structure, the result in Figure 6 surprised us. To generate actors' status trajectories in Figure 6, we returned to the parameter values chosen for Figure 3, with one exception: We shifted β from Eq. (1) from zero to its maximum. We did so by raising the multiplier ρ in Eq. (2). Under this new value of β , a chosen actor's status score now depends significantly on the status scores of those showing her recognition. Thus, for very large values of β , status diffuses extensively through social ties, and an elite's endorsement is particularly influential, much more so than an equivalent flow of recognition from a less prestigious counterpart.

We illustrate the consequence of the top-ranked actor's endorsement of the actor ranked just beneath her in Figure 6. When $\rho = .99$, the Matthew Effect (at the macro level) is no longer realized. While the top-ranked actor starts the contest with a generous lead over the rest of the field, eventually the actor she supports "catches" her in status. Unlike the outcomes shown in Figures 3 and 4, where the Matthew Effect runs its course, the top-ranked actor now impedes her cumulative advantage as a result of endorsing her most proximate, lower-ranked peer. After reaching a global maximum in the fourth round, the top-ranked actor begins her descent just as her peer climbs to meet her in steady-state. Unlike many promotion tournaments—in which "when you win, you go on to the next round" and "when you lose, you lose forever" (Rosenbaum, 1984; Dannefer, 1987, p. 220)—in the contest for status when β is high, the second-ranked actor recovers from her early loss and pulls alongside the market leader. A formalization of this result appears in Appendix D.

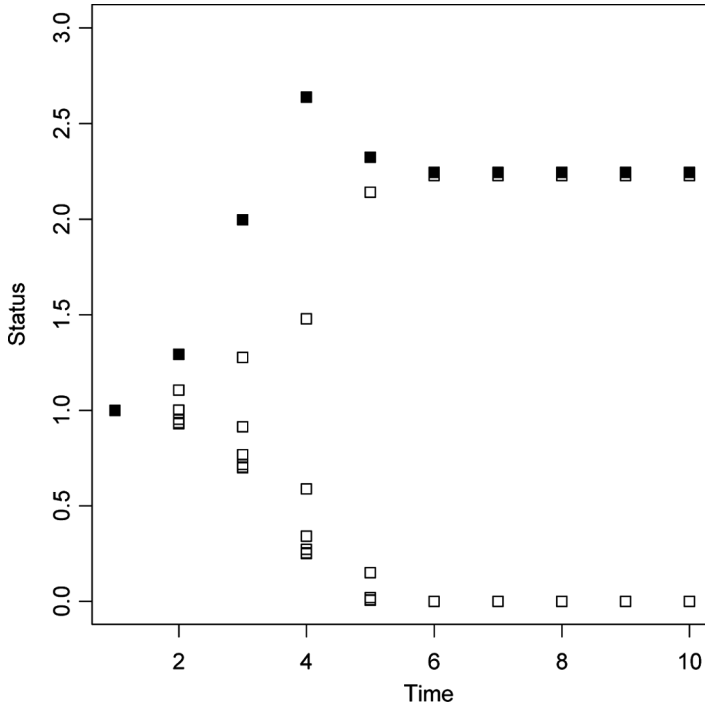


FIGURE 6 The effect of raising β to its maximum value, as a result of setting ρ from Eq. (2) equal to .99. The comparison plot for Figure 6 appears in Figure 3. With the exception of shifting ρ from 0 to .99, all parameter values are identical for Figures 3 and 6.

4. DISCUSSION

Our aim in this article has been to advance our understanding of what reins in the Matthew Effect. Our guiding question has been this: What prevents a high-status actor from securing for herself nearly all the status a social system can supply? We proposed a model in which actors always face a micro-level Matthew Effect. In other words, higher-status actors consistently enjoy more recognition for a given level of quality produced. We then identified the conditions under which the Matthew Effect no longer materializes at the macro level, that is, when the top-ranked actor ceases to dominate the market for intangibles. Stated in our model's formal terms, we found that the Matthew Effect fails when β is high but works when β is low. More generally, the Matthew Effect is curtailed when status

diffuses through social relations but operates freely when actors' status levels are unaffected by the status levels of those giving them recognition.¹⁰

Although the analogy does not hold in all respects, we view physical structures and social structures as similar in an important sense; just as heat can diffuse through, or remain contained by, the floors of a building, status can seep through, or remain sealed off from, the social relations in a network. Correspondingly, in what we will refer to as “porous systems,” as the top-ranked actor endorses the actor just beneath her in the tournament for intangibles, there is a transfer of her status to this lower-ranked counterpart—who can, as a result, eventually draw alongside the top-ranked actor, thereby shutting down the Matthew Effect.

Clearly, our choice to deploy a formal model restricts the generality of this result. Consequently, before noting implications for subsequent empirical investigations, we discuss two central limitations as a way of highlighting the types of observable social networks to which our primary finding applies most directly.

4.1. Limitations

We emphasize first the necessity of status and cost varying inversely. We worked from this inverse relation, setting $c > 1$ in Eq. (5),

¹⁰Our discussion of status diffusion raises the question of how the latter differs from Podolny's (2005) intriguing notion of status leakage: “In my conception, the fundamental check on the Matthew Effect is that high-status actors fear a loss of status due to any association with the low-status actors. Were it not for the risk of their status leaking through exchange relations, the high-status actors would permeate all market niches” (p. 37). Thus, in Podolny's model, a top-ranked actor suffers, or fears suffering, a status decline after affiliating with (and thus being stained by) a lower-ranked actor—for example, an elite company declines in status after associating with (or entering the niche of) a less well-regarded company. In contrast, in our model, a top-ranked individual falls in status after endorsing (and thus abetting the ascent of) a lower-ranked actor—for instance, an elite leader suffers status loss through anointing, and then getting eclipsed by, a lieutenant. The approaches are similar in that status-spillovers occur in both accounts. The approaches differ in three main respects. First, our model more restrictively rests on a concrete endorsement, rather than affiliation or co-presence in the same market niche. Second, loss of status for the top-ranked actor probably occurs over a longer time-horizon in our approach. That is, entering the “wrong” niche may prove immediately status-corrosive, but suffering status loss as an anointed associate ascends may require more time. Third, elite actors are almost certainly more aware of the risk of status loss in Podolny's account than in ours. Put differently, while a firm is generally quite conscious of the penalty of moving down market (and thus may never make the move), in our approach a leader can be blindsided by the status mobility of a chosen follower.

assuming that higher-status actors find it easier to produce at a given level of quality. Typically, as we argued, more prestigious actors get better resources, come across good information faster, attract talented collaborators more easily, and benefit from others' favorable expectations about their future prospects. Wherever this inverse relation between status and cost unravels, however, the intuitions of our model no longer apply. Although we posited $c > 1$, empirical settings certainly exist for which $c < 1$. Costs *increase* with status when, among other things, elites grow complacent, distracted, or both. Unchallenged by direct competition, they can resemble "lazy monopolists" (Hirschman, 1970, pp. 57–75) who face lowered aspirations (Barnett and Hansen, 1996; Bothner, Kang, and Stuart, 2007) and ultimately find it harder to do the work necessary to maintain their status. High-status actors can also get entangled in *more* nonproductive obligations. Acclaimed novelists forced regularly to make public appearances, superstar athletes obliged to keep up with a stream of marketing opportunities, and top academics constantly called upon to write for the popular press are examples of status-based distraction. When status and cost vary positively, our model no longer applies, for it is then that elite actors face unfavorable circumstances inconsistent with the advantages of the micro-level Matthew Effect from which our model proceeds.

We also underscore a second rule on which our model rests: This is the requirement that every actor must recognize, respect, or esteem at least one other individual in the system. We defined flows of recognition broadly and wrote our recognition function so that each actor j has to make allocations from her budget across the other $n - 1$ actors in the system. We therefore worked from the premise that actively "liking *someone*"—or at least appearing to do so—is a prerequisite for participation in the social structure. This supposition follows from much earlier sociological research, such as Gouldner's (1960) discussion of the reciprocity norm and Blau's (1964) notion of the "dilemma of leadership." Blau (1964, p. 203) argued that although elites must maintain some distance from their underlings, they can never achieve complete independence; rather, they have to attend to the task of "earning the social approval" of their subordinates. Without meting out at least a modicum of respect, a high-status actor would no longer qualify for membership in the social system; her perceived

distaste for the group would rob her of legitimacy and thus of her status, at least locally.¹¹

Yet there are some social systems in which individuals who neither initiate nor reciprocate are nonetheless recipients of status-conferring gestures. These systems often reside at society's margins, however. They fit Goffman's (1961) description of "total institutions," such as prisons and mental hospitals, where subordinates' membership is involuntary. Other settings in which recognition flows in just one direction—from follower to leader—include some religious and military organizations. In our model, were it not for the premise that all actors must show respect to at least one other actor, the actor ranked just below the top-ranked actor in the contest could never catch her in status, and the Matthew Effect would always unfold unchecked.¹²

Thus, while the conditions our model requires do not reside in all social domains, they do characterize many. Our primary finding carries implications for settings in which status and cost vary inversely and where all contestants must respond favorably to at least one other member of the system in order to belong to it legitimately.

4.2. Implications for Empirical Research: Porous Versus Insulated Systems

We conclude with a single, falsifiable hypothesis for future empirical investigations that follows immediately from our model's main result—namely, that *the Matthew Effect is constrained in "porous" systems (where status diffuses) but operates freely in "insulated" systems (where status is contained)*. Weber's (1946) ideal-types are especially useful for briefly sketching the most salient differences

¹¹We note that the multiplicative function in Eq. (7) for determining recognition does imply a caveat: If a focal individual were the *only* contestant possessing a positive status score (and thus the sole contestant capable of contributing a non-zero level of quality), this individual, by construction, could not show recognition to anyone. However, more than one contestant ended in steady-state with a non-zero level of status in all simulations.

¹²Going beyond the assumptions of our model, one can certainly imagine (especially in the "total institutions" just mentioned) situations in which the top-ranked actor never dispenses recognition to others, but nonetheless fails to preserve the Matthew Effect because of a lower-ranked actor who gains the support of those ranked even further down and initiates a coup. This possibility is consistent with Luhmann's (1987) preference-based constraint on the Matthew Effect, according to which individuals have strong desire for status mobility and turnover, and, more generally, with research underscoring lower-status actors' heightened openness to social influence (Bothner, 2003; Barnett and Pontikes, 2008).

between the kinds of social systems just introduced. In particular, although correspondence to the ideal-types is necessarily only partial, we see porous systems as closer to the charismatic type and insulated systems as nearer to the bureaucratic type.

Using Weber's models selectively, we thus envision porous systems—those most inimical to the full realization of the Matthew Effect—as marked by the following characteristics: Their elites, like many CEOs credited with advancing a “charismatic orientation” in U.S. businesses (Khurana, 2002, pp. 67–73), are thought to possess exceptional capabilities to influence others, capabilities that enable them to create meaning, anoint followers, and even impart special qualities. Imbued with “genuine charisma,” they possess an “ability to internally generate and externally express extreme excitement, an ability which makes one the object of intense attention and unreflective imitation by others” (Greenfeld, 1985, p. 122).

What is most important, though, is how incumbents of porous systems collectively perceive elites' social ties. We see incumbents construing these elites' connections as conduits out of which almost divine favor emanates. Porous systems, much like the charismatic milieux envisioned by Weber, are social networks that are intensely affective and personal, and whose members place exceptional weight on social (as opposed to just human) capital. As our model suggests, it is in these settings that elites' endorsements of others paradoxically compromise their own chances of accumulating ever-greater status-based advantages. Tying this to our earlier discussion, an implicit understanding of this paradox appears to explain the tendency of corporate leaders to extinguish their heirs-apparent, after having extolled their heirs' abilities in public.

Turning to less familiar terrain, this may also be why dictators often populate their inner circle with those who cannot surpass them in status, such as eunuchs (Cosser, 1964) or women bodyguards (Dowdney, 1998), or regularly sentence those closest to them to exile or death. Consider, as an historical example, Mao Zedong's strategic dealings with Deng Xiaoping: as Deng rose through the Chinese Communist Party on the force of Mao's endorsement, Mao saw that Deng could displace him. Consequently, Mao released propaganda condemning Deng and eventually sent him into exile, where he was forced to repair tractors (Evans, 1995). Yet it was due in part to the initial transfer of status from Mao to Deng that Mao came to view Deng as such a pernicious threat.

Conversely, insulated systems—where we expect cumulative advantage to persist—are those whose elites have no special grace to impart. Accordingly, social ties are not pathways for favor but are only

routes for communication. Far from being intensely affective, these ties are more formal. This not to say that micro-level Matthew Effects fail to occur in insulated systems, however. To the contrary, higher-status actors still get more credit than lower-ranked counterparts for quality produced, and status thus does affect perceptions—but restrictively, in the sense of only shaping perceptions of *output*. What does not occur in insulated systems is *person-to-person* status diffusion. Halos are individually beneficial in insulated systems, but they do not pass from leaders to heirs, and thus chances for macro-level Matthew Effects end up being greater.

Using our sketch of how porous and insulated systems differ, it is not difficult to envision ways to test the assertion that porous systems tend to constrain cumulative advantage, while insulated systems give the Matthew Effect freer course. An especially promising route is to exploit within-firm variation in organizational culture brought on by aging. To the extent that a focal firm is generally closer to the porous (charismatic) type in its nascent phase but nearer to the insulated (bureaucratic) type as it matures, we predict that elites' capacity to enjoy cumulative advantage will rise as a chosen firm moves through its life course. That is, historical case studies should reveal more frequent coups by anointed heirs (and, correspondingly, more ruthless efforts by CEOs to stave them off) when a company is at a fledgling stage—net of the liabilities of newness (Stinchcombe, 1965) that put a sitting CEO at risk. It is in this young phase that we expect the greatest levels of status diffusion from leaders to direct reports, and thus tighter constraints on leaders' opportunities to enjoy the full fruits of the Matthew Effect.

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REFERENCES

- Alexander, J. C. (1987). *Twenty Lectures: Sociological Theory Since World War II*. New York: Columbia University Press.
- Allison, P. D. & Stewart, J. A. (1974). Productivity differences among scientists: evidence for accumulative advantage. *American Sociological Review*, 39, 596–606.

- Ball, W. W. R. (1960). *A Short Account of the History of Mathematics*. New York: Dover Publications.
- Barnard, C. I. (1938). *The Functions of the Executive*. Cambridge, MA: Harvard University Press.
- Barnett, W. P. & Hansen, M. T. (1996). The red queen in organizational evolution. *Strategic Management Journal*, *17*, 139–157.
- Barnett, W. P. & Pontikes, E. (2008). The red queen, success bias, and organizational inertia. *Management Science*, *54*, 1237–1251.
- Benjamin, B. & Podolny, J. M. (1999). Status, quality, and social order in the California wine industry. *Administrative Science Quarterly*, *44*, 563–589.
- Blau, P. M. (1955). *The Dynamics of Bureaucracy: A Study of Interpersonal Relations in Two Government Agencies*. Chicago: University of Chicago Press.
- Blau, P. M. (1964). *Exchange and Power in Social Life*. New York: J. Wiley.
- Bonacich, P. (1987). Power and centrality: a family of measures. *American Journal of Sociology*, *92*, 1170–1182.
- Bothner, M. S. (2003). Competition and social influence: the diffusion of the sixth-generation processor in the global computer industry. *American Journal of Sociology*, *108*, 1175–1210.
- Bothner, M. S., Kang, J., & Stuart, T. E. (2007). Competitive crowding and risk taking in a tournament: evidence from NASCAR racing. *Administrative Science Quarterly*, *52*, 208–247.
- Bothner, M. S., Stuart, T. E., & White, H. C. (2004). Status differentiation and the cohesion of social networks. *The Journal of Mathematical Sociology*, *28*, 261–295.
- Boyer, C. B. (1968). *A History of Mathematics*. New York: J. Wiley.
- Broughton, W. & Mills, Jr., E. W. (1980). Resource inequality and accumulative advantage: stratification in the ministry. *Social Forces*, *58*, 1289–1301.
- Caldini, R. (1989). Indirect tactics of image management: beyond basking. In R. A. Giacalone & P. Rosenfeld (Eds.), *Impression Management in the Organization* (pp. 45–56). Hillsdale, NJ: Lawrence Erlbaum.
- Carlton, J. (1997). *Apple: The Inside Story of Intrigue, Egomania, and Business Blunders*. New York: Times Business.
- Collins, R. (2000). Situational stratification: a micro–macro theory of inequality. *Sociological Theory*, *18*, 17–43.
- Cooney, M. (1994). Evidence as partisanship. *Law and Society Review*, *28*, 833–858.
- Coser, L. A. (1964). The political functions of eunuchism. *American Sociological Review*, *29*, 880–885.
- Dannefer, D. (1987). Aging as intracohort differentiation: accentuation, the Matthew Effect, and the life course. *Sociological Forum*, *2*, 211–236.
- Dasgupta, P. (1995). The population problem: theory and evidence. *Journal of Economic Literature*, *33*, 1879–1902.
- Dowdney, M. (1998, June 12). Girl guard dies saving Gaddafi in gun attack. *The Free Library*. Retrieved February 24, 2010 from <http://www.thefreelibrary.com/Girl-guard-dies-saving-Gaddafi-in-gun-attack.-a060563920>
- Elder, G. H. (1969). Occupational mobility, life patterns, and personality. *Journal of Health and Social Behavior*, *10*, 308–323.
- Evans, R. (1995). *Deng Xiaoping and the Making of Modern China*. London: Penguin.
- Fleming, L., Simcoe, T. S., & Waguespack, D. (2008). *What's in a Name? Status, Discrimination, and the Mathew Effect* (Working Paper). Toronto, Canada: University of Toronto, Rotman School of Business.
- Goffman, E. (1961). *Asylums: Essays on the Social Situation of Mental Patients and Other Inmates*. Garden City, NY: Anchor Books.

- Goode, W. J. (1978). *The Celebration of Heroes: Prestige as a Social Control System*. Berkeley, CA: University of California Press.
- Gouldner, A. W. (1960). The norm of reciprocity: a preliminary statement. *American Sociological Review*, 25, 161–178.
- Greenfeld, L. (1985). Reflections on two charismas. *British Journal of Sociology*, 36, 117–132.
- Hagstrom, W. O. (1965). *The Scientific Community*. New York: Basic Books.
- Hirschman, A. O. (1970). *Exit, Voice, and Loyalty: Responses to Decline in Firms, Organizations and States*. Cambridge, MA: Harvard University Press.
- Homans, G. C. (1961). *Social Behavior: Its Elementary Forms*. New York: Harcourt Brace & World.
- Kanter, R. M. (1977). *Men and Women of the Corporation*. New York: Basic Books.
- Khurana, R. (2002). *Searching for a Corporate Savior: The Irrational Quest for Charismatic CEOs*. Princeton, NJ: Princeton University Press.
- Levitt, B. & Nass, C. (1989). The lid on the garbage can: institutional constraints on decision making in the technical core of college-text publishers. *Administrative Science Quarterly*, 34, 190–207.
- Link, B. & Milcarek, B. (1980). Selection factors in the dispensation of therapy: the Matthew Effect in the allocation of mental health resources. *Journal of Health and Social Behavior*, 21, 279–290.
- Luhmann, N. (1987). The evolutionary differentiation between society and interaction. In J. C. Alexander, B. Giesen, R. Munch, & N. J. Smelser (Eds.), *The Micro-Macro Link* (pp. 112–131). Berkeley, CA: University of California Press.
- Merton, R. K. (1968). The Matthew Effect in science. *Science*, 159, 56–63.
- Merton, R. K. (1988). The Matthew Effect in science, II: cumulative advantage and the symbolism of intellectual property. *Isis*, 79, 606–623.
- Park, R. E. & Burgess, E. W. (1921). *Introduction to the Science of Sociology*. Chicago: The University of Chicago Press.
- Parsons, T. (1963). On the concept of influence. *Public Opinion Quarterly*, 27, 37–62.
- Podolny, J. M. (1993). A status-based model of market competition. *American Journal of Sociology*, 98, 829–872.
- Podolny, J. M. (2001). Networks as the pipes and prisms of the market. *American Journal of Sociology*, 107, 33–60.
- Podolny, J. M. (2005). *Status Signals: A Sociological Study of Market Competition*. Princeton, NJ: Princeton University Press.
- Podolny, J. M. & Phillips, D. J. (1996). The dynamics of organizational status. *Industrial and Corporate Change*, 5, 453–472.
- Ramberg, P. J. (2003). *Chemical Structure. Spatial Arrangement: The Early History of Stereochemistry, 1874–1914*. Aldershot, UK: Ashgate Publishing Company.
- Rao, H. (1994). The social construction of reputation: certification contests, legitimation, and the survival of organizations in the American automobile industry: 1895–1912. *Strategic Management Journal*, 15, 29–44.
- Richards, A. I. (1968). Keeping the king divine. *Proceedings of the Royal Anthropological Institute of Great Britain and Ireland* (pp. 23–35). London: Royal Anthropological Institute of Great Britain and Ireland.
- Rosenbaum, J. E. (1984). *Career Mobility in a Corporate Hierarchy*. Orlando, FL: Academic Press.
- Spence, A. M. (1974). *Market Signaling: Informational Transfer in Hiring and Related Screening Processes*. Cambridge, MA: Harvard University Press.
- Stinchcombe, A. L. (1965). Social structure and organizations. In J. March (Ed.), *Handbook of Organizations* (pp. 142–193). Chicago: Rand McNally.

- Taylor, P. A. (1987). The celebration of heroes under communism: on honors and the reproduction of inequality. *American Sociological Review*, 52, 143–154.
- Walberg, H. J. & Tsai, S. (1983). “Matthew” Effects in education. *American Educational Research Journal*, 20, 359–373.
- Weber, M. (1946). *From Max Weber: Essays in Sociology*. (H. H. Gerth & C. W. Mills, Trans.). New York: Oxford University Press.
- Weiss, Y. (1984). Wage contracts when output grows stochastically: the roles of mobility costs and capital market imperfections. *Journal of Labor Economics*, 2, 155–173.
- White, H. C. (2002). *Markets from Networks: Socioeconomic Models of Production*. Princeton, NJ: Princeton University Press.
- Whyte, W. F. (1993). *Street Corner Society: The Social Structure of an Italian Slum*. Chicago: University of Chicago Press.
- Yang, D. (1990, June). Patterns of China’s regional development strategy. *China Quarterly*, 122, 230–257.
- Zuckerman, E. W. & Kim, T. (2003). The critical trade-off: identity assignment and box-office success in the feature film industry. *Industrial and Corporate Change*, 12, 27–67.

APPENDIX A: GLOSSARY OF SYMBOLS

S_{it}	The time-varying status of the focal actor
α	A scaling constant for the computation of status scores
β	A parameter determining the diffusion of status through social relations
ρ	A multiplier that determines β [ρ ranges from 0 to .99 and acts on the inverse of the largest normed eigenvalue of \mathbf{R}_t ; $\rho = .99 \Leftrightarrow$ The Matthew Effect is constrained; $\rho = 0 \Leftrightarrow$ The Matthew Effect operates freely]
r_{ijt}	The non-normalized flow of recognition from j to i at t
R_{ijt}	The flow of recognition from j to i at t , collected in \mathbf{R}_t
\mathbf{R}_t	The matrix recording pair-wise levels of recognition among actors
$S_{F,i,t+1}$	Future status forecasted by the focal actor
S_{\max}	The maximum status level attainable in the contest
n	The number of actors in the contest
$C_{F,it}$	Cost forecasted by the focal actor as a function of quality
Q_{it}	Time-varying quality contributed by the focal actor
k	A multiplier affecting cost
a	An exponent by which quality Q_{it} is raised
A_i	Time-constant ability of the focal actor
b	An exponent (multiplied by -1) to which ability A_i is raised
c	An exponent (multiplied by -1) to which status S_{it} is raised
ME_{t+1}	The extent to which the Matthew Effect has been realized at the macro-level at $t + 1$ measured as the top-ranked actor’s status over the sum of all status scores

APPENDIX B: DERIVING \mathbf{S}_{\max}

To derive the upper bound on status, shown previously in Eq. (4), we start from three points. First, we note that the measure of status depicted in Eq. (1) may be more fully expressed (see Bonacich, 1987, p. 1173, Eq. (5)) as:

$$\mathbf{S}_i(\alpha, \beta) = \alpha \sum_{k=0}^{\infty} \beta^k \mathbf{R}^{k+1} \mathbf{1} = \alpha (\mathbf{R} \mathbf{1} + \beta \mathbf{R}^2 \mathbf{1} + \beta^2 \mathbf{R}^3 \mathbf{1} + \dots) \quad (\text{B.1})$$

Second, we work from the assumption that the recognition matrix \mathbf{R} that is conducive to a single elite virtually monopolizing available status assumes this form (where we set $n = 3$ for simplicity):

$$\mathbf{R} = \begin{bmatrix} 0 & 1 & 1 \\ 1/(n-1) & 0 & 0 \\ 1/(n-1) & 0 & 0 \end{bmatrix} \quad (\text{B.2})$$

Under this scenario, the top-ranked actor, occupying the first row, secures all the recognition supplied by the other actors, and in turn evenly spreads her outgoing flows of recognition (respect or esteem) across these other actors in the first column.

Third, consistent with our discussion of the recognition function in Section 2.5 of the main text, each of the n actors in the system has 1.00 units of recognition to allocate across the remaining $n - 1$ actors. We imposed this constraint to reflect the zero-sum nature of recognition acts.

Next, to calculate the infinite sum in (B.1), it is necessary to determine which matrices give the various powers of \mathbf{R} . This is straightforward because of the simple periodicity, of order two, for these powers. Every odd power of \mathbf{R} , including the first power shown above, is equal to \mathbf{R} itself. And every even power of \mathbf{R} is given by the following matrix, so that $\mathbf{R}^2 = \mathbf{R}^4 = \mathbf{R}^6$ and so on:

$$\mathbf{R}^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/(n-1) & 1/(n-1) \\ 0 & 1/(n-1) & 1/(n-1) \end{bmatrix} \quad (\text{B.3})$$

With (B.3) as the matrix for any even power of \mathbf{R} , the infinite sum may be calculated from (B.1) as:

$$\mathbf{S}_i(\alpha, \beta) = \alpha \left[\begin{bmatrix} 0 & 1 & 1 \\ 1/(n-1) & 0 & 0 \\ 1/(n-1) & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right]$$

$$+ \beta \cdot \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1/(n-1) & 1/(n-1) \\ 0 & 1/(n-1) & 1/(n-1) \end{array} \right] \cdot \left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right] + \dots \quad (\text{B.4})$$

with further repetition. And expanding the multiplications brings us to:

$$\mathbf{S}_t(\alpha, \beta) = \alpha \left[\left[\begin{array}{c} (n-1) \\ 1/(n-1) \\ 1/(n-1) \end{array} \right] + \beta \cdot \left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right] + \beta^2 \cdot \left[\begin{array}{c} (n-1) \\ 1/(n-1) \\ 1/(n-1) \end{array} \right] + \dots \right] \quad (\text{B.5})$$

At this juncture, we can break the infinite sum into two parts: (i) those containing the even terms, and (ii) those containing the odd. The even terms yield an infinite geometric series with ratio β^2 and sum:

$$\left[\begin{array}{c} (n-1) \\ 1/(n-1) \\ 1/(n-1) \end{array} \right] \cdot \frac{1}{1-\beta^2}$$

The sum of the odd terms is also an infinite geometric series with ratio β^2 and sum:

$$\left[\begin{array}{c} \beta \\ \beta \\ \beta \end{array} \right] \cdot \frac{1}{1-\beta^2}$$

Consequently, the full status levels (leaving aside for now the effect of α) are:

$$\left[\begin{array}{c} (n-1) + \beta \\ 1/(n-1) + \beta \\ 1/(n-1) + \beta \end{array} \right] \cdot \frac{1}{1-\beta^2}$$

In addition, because the second term $\frac{1}{1-\beta^2}$ is common to all entries in the preceding vector, it can be ignored. Before computing S_{\max} the value of the scaling factor α must also be derived. We chose α so that squared length of \mathbf{S}_t equals n (as noted in our discussion of the measurement of status in Section 2.1). The sum of the squares of the n status scores is:

$$\alpha^2 [(n-1+\beta)^2 + (n-1) \cdot (1/(n-1) + \beta)^2]$$

which we can re-express as:

$$\alpha^2 [(n-1+\beta)^2 + (1+\beta \cdot (n-1))^2 / (n-1)]$$

Setting the preceding expression equal to n , solving for α , and simplifying the denominator, we have:

$$\alpha = [(n-1)/(n^2 + (n-1)(\beta^2 + 2\beta - 3))]^{1/2} \quad (\text{B.6})$$

Finally, since status is defined in Eq. (B.1) as the product of the scaling factor α and the infinite sum, for the individual in row 1 of the recognition matrix \mathbf{R} in (B.2), we multiply Eq. (B.6) by the infinite sum for this actor, which we found was $(n-1) + \beta$, in order to compute S_{\max} :

$$S_{\max} = [(n-1)/(n^2 + (n-1)(\beta^2 + 2\beta - 3))]^{1/2}(n-1 + \beta) \quad (\text{B.7})$$

APPENDIX C: DERIVING Q_{it}^*

We begin by setting marginal forecasted status equal to marginal cost:

$$\partial S_{F,i,t+1}(Q_{it})/\partial Q_{it} = \partial C_{F,it}(Q_{it})/\partial Q_{it} \quad (\text{C.1})$$

The left-hand side of (C.1) may be expressed as:

$$\partial S_{F,i,t+1}(Q_{it})/\partial Q_{it} = S_{it} S_{\max} [S_{it} Q_{it} + 1]^{-2} \quad (\text{C.2})$$

and the right-hand side of (C.1) as:

$$\partial C_{F,it}(Q_{it})/\partial Q_{it} = akQ_{it}^{a-1}A_i^{-b}S_{it}^{-c} \quad (\text{C.3})$$

Note that, for $a = 2$, Eq. (C.3) simplifies to:

$$\partial C_{F,it}(Q_{it})/\partial Q_{it} = 2Q_{it}\Theta \quad (\text{C.4})$$

where

$$\Theta = kA_i^{-b}S_{it}^{-c} \quad (\text{C.5})$$

With the results from (C.2) and (C.4) inserted back into (C.1), and after manipulating terms, we have the following cubic equation:

$$Q_{it}^3 + \frac{2}{S_{it}}Q_{it}^2 + \frac{1}{S_{it}^2}Q_{it} - \frac{S_{\max}}{2\Theta S_{it}} = 0 \quad (\text{C.6})$$

Work on cubic equations has shown that (C.6) may be transformed into:

$$y^3 - \frac{1}{3S_{it}^2}y - \left[\frac{S_{\max}}{2\Theta S_{it}} + \frac{2}{27S_{it}^3} \right] = 0 \quad (\text{C.7})$$

where

$$y = Q_{it} + \frac{2}{3S_{it}} \quad (\text{C.8})$$

Furthermore, it is known that the solution to an equation taking the form $y^3 + py + q = 0$ may be expressed as:

$$y = \left[-\frac{q}{2} + \left[\frac{q^2}{4} + \frac{p^3}{27} \right]^{1/2} \right]^{1/3} + \left[-\frac{q}{2} - \left[\frac{q^2}{4} + \frac{p^3}{27} \right]^{1/2} \right]^{1/3} \quad (\text{C.9})$$

where $p = -\frac{1}{3S_{it}^2}$, $q = -\left[\frac{S_{\max}}{2\theta S_{it}} + \frac{2}{27S_{it}^3} \right]$, and $y = Q_{it} + \frac{2}{3S_{it}}$, yielding:

$$Q_{it}^* = \left[-\frac{q}{2} + \left[\frac{q^2}{4} + \frac{p^3}{27} \right]^{1/2} \right]^{1/3} + \left[-\frac{q}{2} - \left[\frac{q^2}{4} + \frac{p^3}{27} \right]^{1/2} \right]^{1/3} - \frac{2}{3S_{it}} \quad (\text{C.10})$$

APPENDIX D: ANALYSIS OF RESULTS

We formally examine two main results in this appendix: the first is an initial finding (depicted in Fig. 5) that the top-ranked actor ascends faster as the cost-related advantages of status increase; the second is our central finding (shown in Fig. 6) that status diffusion counteracts the Matthew Effect. These two results hinge respectively on the parameters c from Eq. (5) and β from Eq. (1).

We start from our cost function in Eq. (5). To make our analyses more tractable, we set the exponent a in (5) equal to 1 rather than 2—the value we had assigned to a previously. Although a quadratic cost function more closely corresponds to what we believe to be characteristic of most status contests, the fact that our expected status function in Eq. (3) is consistently concave in quality Q_{it} guarantees a unique optimal level of quality Q_{it}^* for $a = 1$, and it does so in simpler, more transparent mathematical form. We also believe that generating findings from a linear cost function that agree with results reported in the main body of the text serves as a useful robustness check.

Setting $a = 1$ takes us to an alternative equation for Q_{it}^* (first introduced in Eq. (6)) that is especially intuitive. We again set $\partial S_{F,i,t+1}(Q_{it})/\partial Q_{it}$ from Eq. (3) equal to $\partial C_{F,it}(Q_{it})/\partial Q_{it}$ from Eq. (5). With the exponent a set to unity, equating our two differential equations yields:

$$S_{it} S_{\max} [S_{it} Q_{it} + 1]^{-2} = k A_i^{-b} S_{it}^{-c} \quad (\text{D.1})$$

After rearranging terms and solving for Q_{it} , we express optimal quality Q_{it}^* in Eq. (D.2). This expression reveals almost mandatory patterns for our model's validity, such as optimal quality increasing in the maximum level of status attainable S_{\max} , decreasing in generic costs k , and rising in ability A_i as long as $b > 0$:

$$Q_{it}^* = \frac{[(S_{\max}/k)A_i^b S_{it}^{c+1}]^{1/2} - 1}{S_{it}} \quad (\text{D.2})$$

Using (D.2) we examine the effect of current status on future status, as this effect is realized through our cost function. To proxy future status as simply as possible, recall that we defined status as an aggregation of inflows of recognition and that we expressed the nonnormalized flow of recognition from actor j to actor i at $t + 1$ in Eq. (7) as $r_{ij,t+1} = R_{ijt} \cdot S_{it} \cdot Q_{it}^*$. If we substitute Q_{it}^* from Eq. (D.2) into (7) the result is:

$$r_{ij,t+1} = R_{ijt} [[(S_{\max}/k)A_i^b S_{it}^{c+1}]^{1/2} - 1] \quad (\text{D.3})$$

Taking the derivative of (D.3) with respect to status, Eq. (D.4) shows that future recognition rises in current status iff $c > -1$.

$$\frac{\partial(r_{ij,t+1})}{\partial(S_{it})} = R_{ijt} \cdot \frac{c+1}{2} \cdot [(S_{\max}/k)A_i^b S_{it}^{c-1}]^{1/2} \quad (\text{D.4})$$

This pattern concurs with the basic structure of our model: We certainly expect current status to affect future recognition (and thus future status) positively, but not if status entails the cost disadvantages discussed in Section 4.1.

Our main interest, however, is in whether the positive effect of current status on future recognition rises with the exponent c , which shapes the effect of status on forecasted cost. This second-order effect, if it can be shown, would concur with the result we saw in Figure 5, where the highest-status actor enjoyed a faster climb as c increased from 1 to 10—that is, as the cost advantage of status improved discernibly. Consequently, we differentiate $\frac{\partial(r_{ij,t+1})}{\partial(S_{it})}$ from (D.4) with respect to c . After using the standard differentiation rule for exponentiation together with the product rule, manipulating terms, and then setting $\theta = 1/4 \cdot \sqrt{(S_{\max}/k)A_i^b}$, we have the following mixed partial derivative:

$$\frac{\partial^2(r_{ij,t+1})}{\partial(c)\partial(S_{it})} = R_{ijt} \cdot \theta \cdot S_{it}^{(c-1)/2} (2 + (1+c) \cdot \ln(S_{it})) \quad (\text{D.5})$$

An examination of (D.5) confirms that growth of future recognition with respect to status is enhanced as c rises. This result concurs with

our observation in Figure 5, where the highest-status actor clearly profited from increasing c .

Turning to our second topic in this appendix, we conclude by considering the consequences for this highest-status actor of raising the parameter β to its largest possible value. We saw in Figure 6 that an upward shift β to its maximum curtailed the Matthew Effect as the status scores of the highest and next-ranked actor nearly converged in steady-state. To examine this finding further, we proceed by considering a vector of n status scores ranked such that $S_1 < S_2 < \dots < S_{n-1} < S_n$.

Our objective is to clarify when the actor ranked just beneath the top-ranked actor catches the latter in status, that is, we wish to know when S_n/S_{n-1} descends to 1. Consequently, we imagine further that the top-ranked actor occupies an almost ideal position in the underlying social network in the sense that all other $n - 1$ actors spend their budgets of recognition exclusively on her. That is, if we represent by \mathbf{X} the column-stochastic relational matrix that corresponds to this ideal social network, and allow row n of \mathbf{X} to correspond to S_n , row $n - 1$ to correspond to S_{n-1} , and so on, then in abbreviated (three-by-three) form we have:

$$\mathbf{X} = \begin{bmatrix} 0 & 0 & R_{n-2,n} \\ 0 & 0 & R_{n-1,n} \\ 1 & 1 & 0 \end{bmatrix} \tag{D.6}$$

The top-ranked actor in \mathbf{X} thus enjoys a row sum equal to $n - 1$, and she dispenses recognition to her peers such that $R_{n-1,n} > R_{n-2,n} > \dots > R_{2,n} > R_{1,n}$ and $R_{n-1,n} + R_{n-2,n} + \dots + R_{2,n} + R_{1,n} = 1$.

Using the entries in \mathbf{X} jointly with Eq. (1) for computing status (see also Eq. (B.1) for an expanded version of Eq. (1)) and ignoring the scaling constant α , we can write the status score of the top-ranked actor as:

$$S_n = (n - 1) + \beta \cdot 1 + \beta^2 \cdot (n - 1) + \beta^3 \cdot 1 + \beta^4 \cdot (n - 1) + \dots \tag{D.7}$$

The tractability of (D.7) is an outcome of the convenient features of the powers of \mathbf{X} . The odd powers of \mathbf{X} equal each other—that is, $\mathbf{X} = \mathbf{X}^3 = \mathbf{X}^5$ and so on—and the even powers of \mathbf{X} are also simple replicates of each other, that is, $\mathbf{X}^2 = \mathbf{X}^4 = \mathbf{X}^6$ and et cetera, where:

$$\mathbf{X}^2 = \begin{bmatrix} R_{n-2,n} & R_{n-2,n} & 0 \\ R_{n-1,n} & R_{n-1,n} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{D.8}$$

We can write the status score of the next-highest-ranked actor, fully in keeping with the second rows of \mathbf{X} and \mathbf{X}^2 , as:

$$S_{n-1} = R_{n-1,n} + \beta \cdot (n-1) \cdot R_{n-1,n} + \beta^2 \cdot R_{n-1,n} + \beta^3 \cdot (n-1) \cdot R_{n-1,n} + \beta^4 \cdot R_{n-1,n} \quad (\text{D.9})$$

We can also express (D.7) more simply because it is an infinite sequence which converges to:

$$S_n = (n-1) \cdot \frac{1}{1-\beta^2} + \beta \cdot \frac{1}{1-\beta^2} \quad (\text{D.10})$$

$$S_n = (n-1+\beta) \cdot \frac{1}{1-\beta^2} \quad (\text{D.11})$$

Likewise, we can re-write (D.9) as:

$$S_{n-1} = R_{n-1} \cdot \frac{1}{1-\beta^2} + R_{n-1} \cdot \beta \cdot (n-1) \cdot \frac{1}{1-\beta^2} \quad (\text{D.12})$$

$$S_{n-1} = R_{n-1}(1 + \beta \cdot (n-1)) \cdot \frac{1}{1-\beta^2} \quad (\text{D.13})$$

Our interest finally resides in the conditions under which the actor situated just beneath the top-ranked actor eventually matches the latter in status. Consistent with the time series of status scores shown in Figure 6, we envision $R_{n-1,n}$ getting closer to unity as time passes (i.e., that the top-ranked actor dispenses recognition to the heir-apparent later in the contest). We also assume, consistent with the operation of status diffusion, that ρ from Eq. (2) equals .99, which for a column-stochastic matrix such as \mathbf{X} in (D.6) can be shown to yield $\beta = .99$. Working with the ratio S_n/S_{n-1} from (D.11) to (D.13), we have the function:

$$f(\beta, R_{n-1}) = \frac{n-1+\beta}{R_{n-1}(1+\beta n-\beta)} \quad (\text{D.14})$$

which in turn allows us to finish with:

$$\lim_{\beta, R_{n-1} \rightarrow \{.99, 1\}} f(\beta, R_{n-1}) \approx 1 \quad (\text{D.15})$$